

# Schwarzian derivatives, projective structures, and the Weil-Petersson gradient flow for renormalized volume.

Martin Bridgeman<sup>1</sup>, Jeffrey Brock<sup>2</sup>, Kenneth Bromberg<sup>3</sup>

<sup>1</sup> Boston College

<sup>2</sup> Yale University

<sup>3</sup> University of Utah

We consider complex projective structures and their associated locally convex pleated surface. We relate their geometry in terms of the  $L_2$  and  $L_\infty$  norms the Schwarzian derivative. We show that these give a unifying approach that generalizes a number of well-known results for convex cocompact hyperbolic structures including bounds on the Lipschitz constant for the retract and the length of the bending lamination. We then use these bounds to study the Weil-Petersson gradient flow of renormalized volume on the space  $CC(N)$  of convex cocompact hyperbolic structures on a compact manifold  $N$  with incompressible boundary. This leads to a proof of the conjecture that the renormalized volume has infimum given by one-half the simplicial volume of  $DN$ , the double of  $N$ .