The Graceful Game [1]

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One of the most studied graph labelings is the *graceful labeling*, so named by S. W. Golomb [2] and initially introduced by A. Rosa [3] in 1996. A *graceful labeling* of a graph G with m edges is an injective function $f\colon V(G) \to \{0, 1, \ldots, m\}$ such that, when each edge $uv \in E(G)$ is assigned the *(induced) label* $g(uv) = |f(u) - f(v)|$, all induced edge labels are distinct. Labeling problems are usually studied from the perspective of determining whether a given graph has a required labeling or not. An alternative perspective is to analyze labeling problems from the point of view of combinatorial games. We investigate the Graceful Game, first proposed by Tuza [4]. The *Graceful Game* is defined in the following way: Alice and Bob alternately assign a previously unused label $\phi(v) \in \{0, \ldots, m\}$ to a previously unlabeled vertex v of a given graph G. If both endpoints of an edge $uv \in E(G)$ are labeled, the *label* of uv is defined as $|\phi(u) - \phi(v)|$. A move (label assignment) is said to be *legal* if, after it, all edge labels are distinct. Alice *wins* the game if the whole graph G is gracefully labeled, and Bob *wins* if he can prevent this. In this work, we study winning strategies for Alice and Bob in complete graphs, paths, cycles, complete bipartite graphs, caterpillars, prisms, wheels, hypercubes and powers of paths [1].

References

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