

Renormalization of multicritical circle maps

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We study C^3 orientation preserving circle homeomorphisms with irrational rotation number and non-flat critical points. By a classical result by Yoccoz [12], any two of these maps with same irrational rotation are topologically conjugate. A question that arise is about the conditions to improve the smooth of the conjugacy. In the case of only one critical point, there are many works that had answered this question: joining the works of many authors [1, 3, 2, 6, 7, 9, 8, 10, 11], we know that any two C^4 critical circle map with same irrational rotation number and same odd criticality are conjugate by a C^1 diffeomorphism. Moreover, if their irrational rotation number belongs to a total Lebesgue measure set \mathcal{A} (that contains some Diophantine numbers), then the conjugacy is a $C^{1+\alpha}$ diffeomorphism. For maps with more than one critical point, there is a lot work to do. By [4], we know that for any irrational rotation number the conjugacy that sends critical point to corresponding critical point, (not necessarily preserving criticalities) is a quasisymmetric transformation and therefore is Hölder. In this work, we define the renormalization operator for this kind of maps and assuming some properties of this operator we prove that if the conjugacy preserves criticalities then it is a $C^{1+\alpha}$ diffeomorphism. This result is valid for rotation numbers in \mathcal{A} .

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