

# An approach of the linear heat equation in spaces based on the Fourier transform

Cláudia Aline Azevedo dos Santos Mesquita

Universidade Federal de São Paulo - UNIFESP

Lucas Catão de Freitas Ferreira

Universidade Estadual de Campinas- UNICAMP

april 2019

The aim of this work is to employ a strategy known from fluid dynamics in order to provide results for the linear heat equation  $u_t - \Delta u - V(x)u = 0$  in  $R^n$  with singular potentials. We show well-posedness of solutions, without using Hardy inequality, in a framework based in the Fourier transform, namely,  $PM^k$ -spaces. For arbitrary data  $u_0 \in PM^k$ , the approach allows to compute an explicit smallness condition on  $V$  for global existence in the case of  $V$  with finitely many inverse square singularities. As a consequence, well-posedness of solutions is obtained for the case of the monopolar potential  $V(x) = \frac{\lambda}{|x|^2}$  with  $|\lambda| < \lambda^* = \frac{(n-2)^2}{4}$ . This threshold value is the same one obtained for the global well-posedness of  $L^2$ -solutions by means of Hardy inequalities and energy estimates [6]. Since there is no any inclusion relation between  $L^2$  and  $PM^k$ , our results indicate that  $\lambda^*$  is intrinsic of the PDE and independent of a particular approach. We also analyze the long-time behavior of solutions and show there are infinitely many possible asymptotics characterized by the cells of a disjoint partition of the initial data class  $PM^k$ . This study is published in [2].

## References

- [1] P. BARAS, J. GOLDSTEIN, *The heat equation with a singular potential*, Trans. Amer. Math. Soc. 284 (1984) 121–139.
- [2] L. C. F. FERREIRA, C. A. A. S. MESQUITA, *An approach without using Hardy inequality for the linear heat equation with singular potential*, Communications in Contemporary Mathematics (2015).
- [3] W.M. FRANK, D.J. LAND, R.M. SPECTOR, *Singular potentials*, Reviews of Modern Physics (1971).

- [4] V. FELLI, E. M. MARCHINI, S. TERRACINI, *On Schrödinger operators with multipolar inverse-square potentials*, Journal of Functional Analysis (2007).
- [5] V. FELLI, E. M. MARCHINI, S. TERRACINI, *On Schrödinger operators with multisingular inverse-square anisotropic potentials*, Indiana University Mathematics Journal (2009).
- [6] B. ABDELLAOUI, I. PERAL, V. FELLI, *Existence and multiplicity for perturbations of an equation involving a Hardy inequality and the critical Sobolev exponent in the whole of  $R^n$* , Adv. Differential Equations 9 (2004) 481–508.
- [7] L. C. F. FERREIRA, M. MONTENEGRO, *Existence and asymptotic behavior for elliptic equations with singular anisotropic potentials*, Journal of Differential Equations (2011).