Counting problems and generating functions

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Abstract

To a sequence of integers $a_1, a_2, a_3, \dots$ is associated a generating function: the formal power series $f(x) = \sum a_n x^n$. The generating function provides a way of displaying the sequence. For example, if a_n denotes the number of ways one can write the integer n as a sum of positive integers, the generating function is the partition function $p(x) = \Pi_{m \geq 1}(1 - x^m)^{-1}$, and hence one can compute a_n as the coefficient of x_n in the power series expansion of this function.

The partition function and its generalization the MacMahon function, as well as the generating functions for the Catalan and the Bell numbers, turn up in various geometric settings — triangulations, Schubert calculus, Gromow–Witten and Donaldson–Thomas invariants. In the talk I will give some examples and some hints at explanations.