The Hurwitz curve over a finite fielde and its Weierstrass points for the morphism of lines

Herivelto Borges

(Joint work with N. Arakelian and P. Speziali)

Let $\mathcal{X}$ be an irreducible algebraic curve defined over an algebraically closed field of characteristcs $p \geq 0$. The genus of $\mathcal{X}$ is certainly the most famous birational invariant of $\mathcal{X}$. If $\mathbb{K}(\mathcal{X})$ denotes the function field of $\mathcal{X}$, the group all $\mathbb{K}$-automorphisms of $\mathbb{K}(\mathcal{X})$ is called *automorphism group* of $\mathcal{X}$, and it is denoted by $\mathrm{Aut}(\mathcal{X})$. Such group is another birational invariant of $\mathcal{X}$, and the study of $\mathrm{Aut}(\mathcal{X})$ has become a central problem within the theory of algebraic curves. In this talk, we will consider smooth Hurwitz curves

$$\mathcal{H}_n : XY^n + YZ^n + X^nZ = 0,$$

over the finite field $\mathbb{F}_p$ and provide an explict description of its Weierstrass points for the morphism of lines. That is, we will completely charaterize the special set of points $P \in \mathcal{H}_n$ for which the intersection multiplicity $I(P, \mathcal{H}_n \cap T_P\mathcal{H}_n)$ is somewhat large. As a consequence, the full automorphism group $\mathrm{Aut}(\mathcal{H}_n)$, as well as the genera of all Galois subcovers of $\mathcal{H}_n$ will be presented. In addition, we will discuss how this information can be used to bound the number of $\mathbb{F}_p$-rational points $\mathcal{H}_n$ via Stöhr-Voloch Theory.