Some results about the classification of algebraic group actions in characteristic p

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One of the great achievements of the Lie theory is to have created many beautiful and important relationships between algebraic and geometric objects dependent on continuous parameters and discrete objects of combinatorial nature. This fundamental picture is illustrated in the classification of Chevalley groups in terms of root systems. From this perspective, one may ask to describe not only the group object itself but its transformations on geometric spaces. In the case of reductive group actions, the Luna-Vust theory (1983) gives an answer in the situation that the equivariant birational type is known. A more mysterious issue to elucidate is thereby the case of *actions of non-reductive groups*, where the first prototype to look at is the additive group $\mathbb{G}_a = (k, +)$ of the ground field k. While the abundancy of the $\mathbb{G}_a$-actions for a given finite type scheme plays a key role in affine geometry, in order to have a Lie type correspondence, one needs to rigidify those actions by adding some geometric constraints. A natural condition that appears many times in practice is to consider somehow the $\mathbb{G}_a$-actions that are 'homogeneous' with respect to the grading provided by the action of an algebraic torus. For instance, this recently intervened in the *non-reductive geometric invariant theory* due to Bérczi, Doran, Hawes and Kirwan (2007-2017). A leading and pioneering work that reflects this idea is the one of M. Demazure (1970) who described the automorphism groups of smooth complete toric varieties in terms of the so called *Demazure roots*. In 2003, H. Flenner and M. Zaidenberg classified all the $\mathbb{G}_a$-actions on complex normal affine $\mathbb{C}^\star$-surfaces that are normalized by the $\mathbb{C}^\star$-actions. This latter classification has been extended in higher dimension by A. Liendo (2010) for torus actions with general orbits of codimension one. In this talk, we will present the classification of such $\mathbb{G}_a$-actions over an arbitrary ground field by emphasizing the difference between characteristic 0 and characteristic p and the interaction with the objects of combinatorial nature. This is joint work with A. Liendo.