Stability of non-autonomous difference equations with applications to transport and wave propagation on networks

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In this talk, we address the stability of non-autonomous linear difference equations of the type

$$x(t) = \sum_{i=1}^{m} A_i(t)x(t - L_i), \quad x \in \mathbb{R}^n, \tag{NO-LDE}$$

where the L_i's are the (positive) delays and the matrix-valued functions $A_i(\cdot)$'s are measurable and take values in a bounded set $\mathcal{A}$. We aim at characterizing the asymptotic behavior of solutions of (NO-LDE) in terms of the L_i's and $\mathcal{A}$. We first provide a suitable representation of the solutions in terms of their initial conditions and some time-dependent matrix coefficients. As a consequence, we obtain necessary and sufficient stability criteria for (NO-LDE). In the case of difference equations with arbitrary switching, we obtain a delay-independent generalization of the well-known criterion for autonomous systems due to Hale and Silkowski. We then apply these findings to transport systems and wave propagation on networks with time-varying parameters. We get the following two results: (a) exponential stability of transport systems and wave propagation on networks is robust with respect to variations of the lengths of the edges of the network preserving their rational dependence structure; (b) the wave equation on a network with arbitrarily switching damping at external vertices is exponentially stable if and only if the network is a tree and the damping is bounded away from zero at all external vertices but at most one.
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