Control of PDEs involving boundary layers phenomena

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We consider the controllability of some linear PDEs of the form

$$y_t^\varepsilon + Ay^\varepsilon + \varepsilon By^\varepsilon = 0, \quad \varepsilon > 0$$

where B is an operator with higher order than A. As ε goes to zero, the solution y^ε exhibits singular layers (boundary or internal) which may destroy the underlying controllability property. The main aim of the talk is to discuss the influence of singular phenomena on controllability properties. We shall notably discuss the system

$$\begin{cases} y_{tt}^\varepsilon + \varepsilon y_{xxxx}^\varepsilon - y_{xx}^\varepsilon = 0, & (x,t) \in (0,1)\times(0,T), \\ y^\varepsilon(0,\cdot) = y^\varepsilon(1,\cdot) = y_x^\varepsilon(0,\cdot) = 0, y_x^\varepsilon(1,\cdot) = v^\varepsilon, & t \in (0,T), \\ (y^\varepsilon(\cdot,0), y_t^\varepsilon(\cdot,0)) = (y^0, y^1), & x \in (0,1). \end{cases}$$

(which models the transversal displacement of an homogeneous beam with cross section ε) and determine rigorously an expansion of the null control v^ε of minimal $L^2(0,T)$ norm as follows:

$$v^\varepsilon = \varepsilon^{-1/2} v^0 + v^1 + \varepsilon^{1/2} v^2 + \cdots$$

in term of Dirichlet control functions v^i for nonhomogeneous wave equations.

References

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[2] Castro, Carlos and Münch, Arnaud, *Singular asymptotic expansion of the exact control for the Rayleigh beam with respect to the thickness*, Submitted.