

Analogues of the Brauer-Siegel theorem in arithmetic geometry

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We will explain analogies between the classical Brauer-Siegel theorem, a statement relating asymptotically the class number, regulator of units and discriminant of a number field, and similar statement involving arithmetic invariants of algebraic varieties over a finite or global field. We present precisely the analogy for surfaces over a finite field and for abelian varieties over a global field (i.e. a number field or the function field of a curve over a finite field), surveying some recent results [1, 2, 3, 4]. The proof of Brauer-Siegel theorem relies on the class number formula and analytical estimates for the Dedekind zeta function, the analogy draws on formulae predicted by the Birch & Swinnerton-Dyer conjecture, (resp. Artin-Tate conjecture) and analytical estimates for the relevant L -series. If time permits, we will also formulate a quite general question along these lines, for algebraic varieties over a global field, and develop the case of projective hypersurfaces.

References

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