Bounds on vector fields: degrees and generators

M. Chardin[1], **S. Hamid Hassanzadeh** [2], C. Polini[3] , A. Simis[4], B. Ulrich[5]

[1] Sorbonne University

[2] Federal University of Rio de Janeiro

[3] University of Notre-Dame

[4] Federal University of Pernambuco

[4] Purdue University

Finding algebraic integrals of a vector field is a fascinating question. Over a century ago Poincare had been interested in this question. Besides this fact, it is not clear what he might have asked about the algebraic integrals! The question of finding the minimum degree of a vector field which leaves a variety invariant has had significant progress in the recent years. In this talk which is a report on ongoing work, we present a Commutative Algebraic point of view to the object. We show that the a-invariant of a ring is the numerical invariant which can unify and explain several previous results such as some in [1, 2, 3, 4]. We determine lower bounds, upper bounds and bounds on the number of the generators of the module of non-trivial vector fields which leave a curve invariant.

References

[1] D. Cerveau and A. Lins Neto, *Holomorphic foliations in $\mathbb{P}^2_{\mathbb{C}}$ having an invariant algebraic curve*, Ann. Inst. Fourier **41** (1991), 883–903.

[2] E. Esteves, *The Castelnuovo-Mumford regularity of an integral variety of a vector field on projective space*, Math. Res. Lett. **9** (2002), 1–15.

[3] M. Soares, *The Poincaré problem for hypersurfaces invariant by one-dimensional foliations*, Invent. Math. **128** (1997), 495–500.

[4] M. Soares, *Projective varieties invariant by one-dimensional foliations*, Ann. of Math. **152** (2000), 369–382.