An averaging principle in homogeneous spaces

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Consider an stochastic differential equation on a space such that the trajectories lay on compact surfaces (say, energy levels, foliation etc), depending on the initial condition. We investigate the effective behaviour on the transversal direction of a small transversal perturbation of order ϵ. An average principle has been shown such that the energy level behaviour converges to the solution of a deterministic ODE, according to the average of the perturbing transversal vector field as ϵ goes to zero. Many applications appear in this context. We are particularly interested on the case of a Lie foliation: the leaves are the cosets of a Lie subgroup H in a Lie group G. The interesting phenomenon here is that if the Lie brackets of the transversal perturbation and the vector fields in the SDE do not vanish, noise in the foliated direction are transmitted to the transversal direction, identified with the homogenous space G/H.