Variational formulas for functionals of the Fractional Brownian Motion and applications to Large Deviations Principles

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The equivalence between the large deviations principle (LDP for short) and the Laplace-Varadhan principle in the setting of Polish spaces is the starting point for the designated weak convergence approach to large deviations theory. Instead of the usual approximation procedures and the cumbersome verification of exponential tightness, this approach allows the user to derive simpler sufficient conditions for establishing LDPs that rely on the verification of tightness for the laws of the processes involved and the verification of the convergence, through compactness arguments, in well-known functional spaces, of the associated controlled equations to the problem of finding the rate function for the LDP. This approach was developed in different settings by Dupuis, Ellis, Budhiraja and collaborators (we refer the reader to the book [4], [2] and [3]). It is our purpose to derive a variational formula for functionals of Fractional Brownian Motions (fBMs for short) and to establish a sufficient condition for the verification of a LDP for families of measurable maps of fBMs. As a first application we show how the robustness of this approach allow us to study the first exit time problem of randomly perturbed dynamical systems by the fBM in the small noise limit, bypassing the usual Freidlin-Wentzell toolbox, that is heavily based on the strong Markov property (we refer the reader to the classic reference [5] and the thesis [6], where the author studies the first exit time problem in the spirit of the Freidlin-Wentzell theory for the Lévy case). As a second application we generalize the work [1] giving a nonlinear Feynman-Kac formula for nonlocal partial differential equations (PDEs for short) associated to forward-backward stochastic differential equations driven by the fBM and studying, via probabilistic arguments that rely on Malliavin calculus for those objects, the homogenization regime of those PDEs. This is a joint work with Pedro Catuogno from IMEEC-UNICAMP.

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