

Prescribing the Gauss curvature of hyperbolic convex bodies

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The Gauss curvature measure of Euclidean convex body is a measure on the unit sphere which extends the notion of Gauss curvature to non-smooth bodies. Given a measure μ , Alexandrov's problem consists in finding a convex body whose curvature measure is μ . In Euclidean space, A.D. Alexandrov gave necessary and sufficient conditions on μ for this problem to have a solution, and it was observed later that proving the existence of a convex body of given curvature measure μ is equivalent to an optimal transport problem on the sphere.

In this talk I will address Alexandrov's problem for arbitrary convex bodies in the hyperbolic space. After defining the curvature measure, I will give the necessary and sufficient conditions on a measure μ to solve Alexandrov's problem in this setting, and I will explain how the optimal transport approach leads to a non-linear Kantorovich problem on the sphere, which is the weak form of a Monge-Ampère equation. Joint work with Jérôme Bertrand.