

Mean Fields Games on networks

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We consider Mean Field Games (MFG) for which the state space is a network. More precisely, we consider a coupled system of a backward in time Hamilton-Jacobi-Bellman (HJB) equation and a forward in time Fokker-Planck (FP) equation. The solution u of the HJB equation is the value function of the optimal control problem for a representative agent and the solution m of the Fokker-Planck equation is the density of the distribution of the players. The main difficulty is to deal with the transition conditions at the vertices of the network: The function u is continuous and satisfies general Kirchhoff conditions at the vertices whereas the density m satisfies dual transmission conditions and is discontinuous in general. We will discuss the well-posedness of the system in two cases. The first one is concerned with elliptic systems related to infinite stochastic MFG and the second one addresses parabolic problems related to finite horizon stochastic MFG.