# Existence and regularity of an optimal shape for the Laplacian with a drift

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Let $D \subset \mathbb{R}^d$ be a fixed domain, $m \in (0, |D|)$ and $\tau \geq 0$. For all quasi-open sets $\Omega \subset D$ and all vector fields $V \in L^\infty(\Omega, \mathbb{R}^d)$, let $\lambda_1(\Omega, V)$ be the principal eigenvalue of the operator $L = -\Delta + V \cdot \nabla$ in $\Omega$ under Dirichlet boundary condition. We prove that the following minimization problem:

$$\min \Big\{ \lambda_1(\Omega, V) \ : \ \Omega \subset D \text{ quasi-open}, \ |\Omega| \leq m, \ \|V\|_{L^\infty} \leq \tau \Big\}$$

has a solution. If $V$ is furthermore assumed to be the gradient of a Lipschitz function, we describe the regularity of optimal domains.