Reductions between certain incidence problems and the Continuum Hypothesis

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In this work, we consider two families of incidence problems, $\mathcal{C}_1$ and $\mathcal{C}_2$, which are related to real numbers and countable subsets of the real line. Instances of problems of $\mathcal{C}_1$ are as follows: given a real number x, pick randomly a countable set of reals A hoping that $x \in A$, whereas instances of problems of $\mathcal{C}_2$ are as follows: given a countable set of reals A, pick randomly a real number x hoping that $x \notin A$. One could arguably defend that, at least intuitively, problems of $\mathcal{C}_2$ are easier to solve than problems of $\mathcal{C}_1$. After some suitable formalization, we prove (within **ZFC**) that, on one hand, problems of $\mathcal{C}_2$ are, indeed, at least as easy to solve as problems of $\mathcal{C}_1$. On the other hand, the statement "Problems of $\mathcal{C}_1$ have the exact same complexity of problems of $\mathcal{C}_2$" is shown to be an equivalent of the Continuum Hypothesis (**CH**).