

Uncertainty Principles and Sphere Packings

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In this talk we describe uncertainty principles connected with sphere packings. If both f and \widehat{f} are real and eventually nonnegative, both having nonpositive total mass, then it is impossible for both f and \widehat{f} be nonnegative outside an arbitrarily small neighborhood of the origin. This uncertainty principle (now we call +1 uncertainty) was discovered first in 2010 by Bourgain, Clozel and Kahane with applications to discriminant bounds in algebraic number theory. Recently, in joint work with Henry Cohn, we discovered a cousin uncertainty principle (now called -1 uncertainty) which is connected with bounds sphere packings densities. Adapting the modular functions technique introduced by Viazovska's solution of the 8 and 24 dimensional packing problem we were able to solve completely the +1 uncertainty principle in dimension 12, and thus producing for the first time an extremizer of a uncertainty inequality via modular forms. Time permitting, we will also quickly discuss some unknown numerical phenomena and open conjectures connected with the topic.