

Uniform boundedness principles for Sobolev maps into manifolds

Antonin Monteil ¹

¹ Université Catholique de Louvain

We will discuss the existence of uniform bounds on the energy associated to the problems of weak-bounded approximation, lifting, extension (etc. . .) in $W^{s,p}(\mathcal{M}, \mathcal{N})$, where \mathcal{M}, \mathcal{N} are given Riemannian manifolds, $p \in [1, +\infty)$ and $s \in (0, 1]$. Namely, in some regimes, including the subcritical case $sp < \dim(\mathcal{M})$, we will see that each of the preceding qualitative properties implies a corresponding quantitative energy bound. For instance, if every map $u : \mathcal{M} \rightarrow \mathbb{S}^1$ lifts to \mathbb{R} , then it is always possible to find at least one lifting with Sobolev seminorm controlled linearly by that of u . This can be seen as a nonlinear counterpart of the Banach-Steinhaus theorem.