Representations of n-Lie algebras and twisted algebraic structures

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The notion of n-Lie algebra was introduced by Filippov in [2]. Ternary Lie algebras are related to Nambu mechanics [6], generalizing Hamiltonian mechanics by using more than one hamiltonian. The algebraic formulation of this theory is due to Takhtajan [7]. Moreover, 3-Lie algebras appeared in String Theory, Fuzzy sphere (noncommutative space)and in the context of Bagger-Lambert-Gustavsson model of multiple M2-branes.

In this talk, we discuss a new approach to representation theory of 3-Lie algebras. We define generalized representations of a 3-Lie algebra $\mathfrak{g}$ on a vector space V via canonical structures in the differential graded Lie algebra associated to $\mathfrak{g} \oplus V$. We show that they lead also to a new 3-Lie algebra, which we call a generalized semidirect product. We also develop the corresponding cohomology theory. As application, we study abelian extensions of 3-Lie algebras and provide several examples. If time allows, we will also discuss twisted n-Lie algebras and their representations.

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