Constructions of full diversity rotated D_n-lattices via number fields[1]

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In this talk we will approach algebraic constructions of full diversity rotated D_n-lattices via number fields and their minimum product distances. Diversity and minimum product distance are important lattice parameters related to signal transmission over Rayleigh fading channels [3].

Let $\mathbb{K}$ be a number field of degree n, $\mathcal{O}_{\mathbb{K}}$ its ring of integers and $\alpha \in \mathcal{O}_{\mathbb{K}}$ a totally real and totally positive element. In [1, 2] it was introduced a twisted embedding $\sigma_\alpha : \mathbb{K} \to \mathbb{R}^n$ such that if $\mathcal{I} \subseteq \mathcal{O}_{\mathbb{K}}$ is a free $\mathbb{Z}$-module of rank n, then $\sigma_\alpha(\mathcal{I})$ is a lattice in $\mathbb{R}^n$. It was shown that if $\mathbb{K}$ is a totally real number field, then $\sigma_\alpha(\mathcal{I})$ is a full diversity lattice.

Let $r \geq 3$ be an integer. Consider p, p_1, p_2 prime numbers with $p \geq 7$, $p_1 \geq 5$, $p_2 \geq 5$ and $p_1 \neq p_2$. In order to get full diversity rotated D_n-lattices, constructions of D_n-lattices via twisted embeddings applied to free $\mathbb{Z}$-modules of rank n contained in the totally real number fields $\mathbb{K}_1 = \mathbb{Q}(\zeta_p + \zeta_p^{-1})$, $\mathbb{K}_2 = \mathbb{Q}(\zeta_{2^r} + \zeta_{2^r}^{-1})\mathbb{Q}(\zeta_p + \zeta_p^{-1})$ and $\mathbb{K}_3 = \mathbb{Q}(\zeta_{p_1} + \zeta_{p_1}^{-1})\mathbb{Q}(\zeta_{p_2} + \zeta_{p_2}^{-1})$ were presented in [5, 6]. If it were possible to construct such rotated D_n-lattices via principal ideals of $\mathcal{O}_{\mathbb{K}_i}$, for $i = 1, 2, 3$, their minimum product distances would be twice those obtained in such constructions. However, in [6] it was shown that it is impossible to construct these lattices via fractional ideals of $\mathcal{O}_{\mathbb{K}_i}$, for $i = 1, 2, 3$, and it was presented a necessary condition for constructing a rotated D_n-lattice via a fractional ideal in a totally real Galois extension $\mathbb{K}|\mathbb{Q}$. In particular, if $\mathbb{K}|\mathbb{Q}$ is a totally real

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Galois extension of degree $n \notin \{1, 2, 4\}$ with odd discriminant, then it is impossible to construct a rotated D_n-lattice via a fractional ideal of $\mathcal{O}_{\mathbb{K}}$. Recently, in [4] full diversity rotated D_n-lattices were obtained via free $\mathbb{Z}$-modules contained in subfields of $\mathbb{Q}(\zeta_p + \zeta_p^{-1})$, p prime, for all odd number n.

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