Regularity and rate of approximation for obstacle problems for a class of integro-differential operators

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We consider obstacle problems for three nonlocal operators:

- The integral fractional Laplacian
- The integral fractional Laplacian with drift
- A second order elliptic operator plus the integral fractional Laplacian

For the solution of the problem in the first case, we derive regularity results in weighted Sobolev spaces, where the weight is a power of the distance to the boundary. For the second and third cases we derive, via a Lewy-Stampacchia type argument, regularity results in standard Sobolev spaces. We use these regularity results to derive error estimates for finite element schemes. The error estimates turn out to be optimal in the first case, whereas there is a loss of optimality in the other ones, depending on the order of the integral operator.