Computational Math Meets Geometry

Douglas N. Arnold [1]

[1] University of Minnesota

One of the joys of mathematical research occurs when seemingly distant branches of math come together. A beautiful example occurred over the last decade with the development of the field of compatible, or structure-preserving, discretizations of differential equations, in which ideas from topology and geometry have come to play a key role in numerical analysis. Very roughly, instead of applying standard all-purpose algorithms, such as Runge-Kutta methods and linear multistep methods for ODEs and finite difference or finite element methods for PDEs, far better results can be obtained for various classes of problems by constructing discretization methods which exactly preserve key geometric structures underlying the equations under consideration. Such structures include, for ordinary differential equations, symplecticity, symmetry, invariants and constraints, and, for partial differential equations, de Rham and other cohomologies and associated Hodge theory. We will tour this burgeoning field, demonstrating some of the advances in numerical methods made possible by the new geometrical and topological approaches, and even present a case where the numerical point of view has enabled the resolution of a long open question in algebraic topology.