Renormalizing KPZ equation at weak disorder in dimension ≥ 3

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We study the solution $h_\epsilon(t,x)$ of the (regularized) Kardar-Parisi-Zhang (KPZ) equation on $[0,\infty)\times\mathbb{R}^d, d\geq 3$,

$$\frac{\partial}{\partial t} h_\epsilon = \frac{1}{2}\Delta h_\epsilon + \left[\frac{1}{2}|\nabla h_\epsilon|^2 - C_\epsilon\right] + \beta\epsilon^{\frac{d-2}{2}}\xi_\epsilon \tag{1}$$

with $h_\epsilon(0,x) = 0$. Here $\beta > 0$ quantifies the disorder strength, $\xi_\epsilon = \xi \star \phi_\epsilon$ is a spatially smoothened (at scale ϵ) Gaussian space-time white noise and C_ϵ is a divergent constant as $\epsilon \to 0$.

For sufficiently small β, there exists a stationary solution of (1) for $\epsilon = 1$, and we denote by $h_\epsilon(t,x)$ the one driven by the diffusively rescaled (at scale ϵ), time-reversed and spatially translated white noise $\xi^{\epsilon,t,x}$. Then, it is known that $h_\epsilon(t,x) - \mathfrak{h}_\epsilon(t,x) \to 0$ in probability as $\epsilon \to 0$.

In this talk, we will show convergence in law of the process:

$$\epsilon^{1-\frac{d}{2}}\left[h_\epsilon(t,x) - \mathfrak{h}_\epsilon(t,x)\right] \longrightarrow \mathcal{H}(t,x)$$

to a centered Gaussian field which is the (real-valued) solution of the *non-noisy heat equation* $\partial_t \mathcal{H} = \frac{1}{2}\Delta\mathcal{H}$ with a random initial condition $\mathcal{H}(0,x)$ given by a Gaussian free field on $\mathbb{R}^d$.