Controllability of difference equations

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Difference equations are useful tools in the analysis of some hyperbolic PDEs, in particular systems of PDEs on networks, since they provide a handy representation of some simple dynamics. In this talk, we analyze the controllability of a linear difference equation with finitely many delays.

Three notions of controllability are considered: relative, approximate, and exact. After providing the corresponding definitions, we start by presenting a relative controllability criterion expressed in terms of some coefficients computed inductively from the matrices defining the system. Issues such as the minimal controllability time and the effects of the rational dependence relations of the delays are also discussed.

We then provide a complete characterization of approximate and exact controllability in L^2 for 2-dimensional systems with two delays and a scalar control, which corresponds to the first non-trivial situation. It illustrates most difficulties in the analysis and the subtleties of the controllability criterion one obtains. We also relate these controllability properties to approximate and exact controllability to constants.

Our approach relies on a suitable representation formula for solutions, which had already been used in a previous work in the stability analysis of difference equations. Part of the results of this talk were obtained in collaboration with Y. Chitour and M. Sigalotti.