

A half-space theorem for graphs of constant mean curvature $0 < H < \frac{1}{2}$ in $\mathbb{H}^2 \times \mathbb{R}$

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We study a half-space problem related to graphs in $\mathbb{H}^2 \times \mathbb{R}$, where \mathbb{H}^2 is the hyperbolic plane, having constant mean curvature H defined over unbounded domains in \mathbb{H}^2 .

More precisely, we consider graphs of functions u defined in a domain $D \subset \mathbb{H}^2$ whose boundary ∂D is composed of complete arcs $\{A_i\}$ and $\{B_j\}$, such that the curvatures of the arcs with respect to the domain are $\kappa(A_i) = 2H$ and $\kappa(B_j) = -2H$. These graphs will have constant mean curvature and u will assume the value $+\infty$ on each A_i and $-\infty$ on each B_j . These domains D will be called Scherk type domains and the functions u Scherk type solutions. The existence of these graphs is assured by A. Folha and S. Melo in [2]. In this context, we prove the following result in [1].

Theorem(Wanderley, Mazet). Let $D \subset \mathbb{H}^2$ be a Scherk type domain and u be a Scherk type solution over D (for some value $0 < H < \frac{1}{2}$). Denote by $\Sigma = \text{Graph}(u)$. If S is a properly immersed CMC H surface contained in $D \times \mathbb{R}$ and above Σ , then S is a vertical translate of Σ .

References

- [1] L. MAZET AND G.A. WANDERLEY , *A half-space theorem for graphs of constant mean curvature $0 < H < \frac{1}{2}$ in $\mathbb{H}^2 \times \mathbb{R}$* , Illinois J. of Math. No1, 43-53 2015.
- [2] A. FOLHA AND S. MELO , *The Dirichlet problem for constant mean curvature graphs in $\mathbb{H}^2 \times \mathbb{R}$ over unbounded domains*, Pacific journal of mathematics, v. 251, n. 1, p. 37-65, 2011.