Special groups and quadratic forms over rings with non zero-divisor coefficients

F. Miraglia *

In all that follows, the word **ring** will stand for a reduced (0 is its only nilpotent element), unitary, semi-real (-1 is not a sum of squares) commutative ring, in which 2 is a unit. Recall that a formula in a first-order language with equality is **Horn-geometric** if it is the negation of an atomic formula or the form $\forall \overline{v}(\varphi(\overline{v}) \rightarrow \psi(\overline{v}))$, where φ and $\psi$are **primitive positive (pp)**, that is, of the form $\exists \overline{y}\ \theta(\overline{y}; \overline{v})$, where θ is a conjunction of atomic formulas.

In [1] and [2], it is shown that there is a set of Horn-geometric axioms so that if a p-ring $\langle R, P \rangle$, where P is either proper preorder of the ring R or $P = R^2$, then there is a special group, $G := G_P(R)$, canonically associated to $\langle R, P \rangle$, so that both P-isometry and P-representation of diagonal quadratic forms with unit coefficients in R is faithfully coded by the corresponding concepts in G. It is then shown that a very significant class of preordered rings satisfy this axioms, that in turn yields significant information on the properties of ring-theoretic representation and isometry of these types of quadratic forms (see [1] and [2] for more details).

The present talk reports on joint work with M. Dickmann (Institut de Mathématiques de Jussieu − Paris Rive Gauche (IMJ-PRG))and Hugo Ribeiro (IME-USP), endeavoring to extend the results in [1] and [2] to diagonal quadratic forms over preordered rings, whose coefficients are *non zero-divisors*. We show that also in this case there are Horn-geometric axioms so that if a p-ring, $\langle R, P \rangle$, satisfies these axioms, then there is a special group $G_{\mathcal{N}P}(R)$, so that ring-theoretic P-representation and P-isometry of these forms is faithfully coded by the corresponding concepts in $G_{\mathcal{N}P}(R)$. We then prove that if X is a completely regular topological space, both the ring of bounded real valued continuous functions and the full ring of real valued continuous functions on X satisfy these axioms. The perspective is to extend these results to even wider classes of rings (e.g., reduced f-rings, rings with many units, Archimedean bounded inversion rings, among others), as well as to relate the present pursuit to the results in [1] and [2].

References:

[1] M. Dickmann, F. Miraglia, **Faithfully Quadratic Rings**, Memoirs of the Amer. Math. Soc **1128**, Providence, R.I., November 2015.

[2] M. Dickmann, F. Miraglia, *Faithfully Quadratic Rings; a summary of results*, Banach Center Publ., Inst. Math., Polish Acad. Sci. vol. **106** (2017), 37 - 48.

*Conference proposal for the Brazil-France Logic Meeting, IMPA, July, 2019