On the Determination of the Lévy Exponent in Asset Pricing Models

Lane Hughston[1]

[1] Goldsmiths College, University of London

We consider the problem of determining the Lévy exponent in a Lévy model for asset prices given the price data of derivatives. The model, formulated under the real-world measure $\mathbb{P}$, consists of a pricing kernel $\{\pi_t\}_{t\geq 0}$ together with one or more non-dividend-paying risky assets driven by the same Lévy process. If $\{S_t\}_{t\geq 0}$ denotes the price process of such an asset then $\{\pi_t S_t\}_{t\geq 0}$ is a $\mathbb{P}$-martingale. The Lévy process $\{\xi_t\}_{t\geq 0}$ is assumed to have exponential moments, implying the existence of a Lévy exponent $\psi(\alpha) = t^{-1} \log \mathbb{E}(\mathrm{e}^{\alpha \xi_t})$ for α in an interval $A \subset \mathbb{R}$ containing the origin as a proper subset. We show that if the initial prices of power-payoff derivatives, for which the payoff is $H_T = (\zeta_T)^q$ for some fixed time $T > 0$, are given for a range of values of q, where $\{\zeta_t\}_{t\geq 0}$ is the so-called benchmark portfolio defined by $\zeta_t = 1/\pi_t$, then the Lévy exponent is fully determined, up to an irrelevant linear term. In such a setting, derivative prices embody complete information about asset price jumps: in particular, the spectrum of the asset price jumps can be worked out from current market prices of derivatives. More generally, if $H_T = (S_T)^q$ for a general non-dividend-paying risky asset driven by a Lévy process, and if we know that the pricing kernel is driven by the same Lévy process, up to a constant factor of proportionality, then from the current prices of power-payoff derivatives we can infer the structure of the Lévy exponent up to a transformation of the form $\psi(\alpha) \to \psi(\alpha + \mu) - \psi(\mu) + c\alpha$, where c and μ are constants. (Based on work carried out in collaboration with George Bouzianis, Goldsmith's College, University of London.)