On the Joint Calibration of SPX and VIX Options

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Since VIX options started trading in 2006, many researchers have attempted to build a model for the SPX that jointly calibrates to SPX and VIX options. In 2008, Jim Gatheral showed that a diffusive model could approximately, but not exactly, fit both markets. Later, others have argued that jumps in the SPX were needed to jointly calibrate both markets. We revisit this problem, asking the following questions: Does there exist a continuous model on the SPX that jointly calibrates to SPX options, VIX futures, and VIX options? If so, how to build one such model? If not, why? We present a novel approach based on the SPX smile calibration condition $E[\sigma_t^2|S_t] = \sigma_{lv}^2(t, S_t)$. In the limiting case of instantaneous VIX, the answers are clear and involve the timewise convex ordering of two distributions (local variance and instantaneous variance) and a novel application of martingale transport to finance. The real case of a 30-day VIX is more involved, as time-averaging and projection onto a filtration can undo convex ordering. We show that in usual market conditions, for short maturities, the distribution of VIX^2 in the local volatility model is NOT smaller than the market-implied distribution of VIX^2 in convex order, and we explain that fast mean-reverting volatility models and rough volatility models are able to reproduce this surprising behavior. In particular, we prove two conjectures to be wrong: among all continuous models on the SPX calibrated to the SPX smile, the local volatility model does NOT produce the maximum value of VIX futures, nor does it produce the minimum value of options on VIX2.