REFINED TROPICAL CURVE COUNTING

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Let S be a smooth toric surface, and let L be a line bundle on S. With Block we used tropical geometry to define refined Severi degrees $N_{(S,L),\delta}(y)$ which are polynomials in y interpolating between the Severi degrees counting complex δ-nodal curves and the Welschinger invariants counting real curves. If L is sufficiently ample, they are conjectured to be equal to the refined invariants $N_\delta^{(S,L)}(y)$ defined with Shende in terms of Hilbert schemes of points, relating them to refined Donaldson-Thomas invariants. We will review these definitions and results about these invariants

In the second part of the talk we will briefly review newer results (by other people):

(1) The approach of Nicaise-Payne-Schroeter to prove the conjectured equality between the refined invariants and the refined Severi degrees using nonarchimedian motivic volume.
(2) The interpretation of Bousseau of refined Severi degrees as logarithmic Gromov-Witten invariants with λ-classes.