What is a tropical plane curve?

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Brodsky, Joswig, Morrison and Sturmfels showed that not all abstract tropical curves of genus 3 can be realized as a tropicalization of a quartic in $\mathbb{R}^2$. We focus on the interior of the maximal cones in the moduli space and study all curves which can be realized as a faithful tropicalization in a tropical plane. Reflecting the algebro-geometric world, these are exactly those which are not realizably hyperelliptic. Our approach is constructive: For any not realizably hyperelliptic curve, we explicitly construct a realizable model of the tropical plane and a faithfully tropicalized quartic in it. These constructions rely on modifications resp. tropical refinements. Conversely, we prove that any realizably hyperelliptic curve cannot be embedded in such a fashion. For that, we rely on the theory of tropical divisors and embeddings from linear systems, and recent advances in the realizability of sections of the tropical canonical divisor.