On the biregular geometry of the Fulton-MacPherson compactification

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The Fulton-MacPherson configuration space $X[n]$ is a natural compactification of the configuration space of n ordered points on a smooth projective variety X.
The Kontsevich moduli space $\overline{M}_{0,n}(\mathbb{P}^N, d)$ parametrizing stable maps from n-pointed rational curves to a projective space is another widely studied algebraic variety, and plays a central role in algebraic geometry, string theory and Gromov-Witten theory.
These two spaces are related by an isomorphism $\mathbb{P}^1[n] \cong \overline{M}_{0,n}(\mathbb{P}^1, 1)$. Furthermore, the Fulton-MacPherson configuration space $C[n]$ of n ordered points in a smooth projective curve C is closely related to the Deligne-Mumford compactification $\overline{M}_{g,n}$ of the moduli space of smooth curves of genus g with n-marked points. Indeed, $\overline{M}_{0,n}$ is a GIT quotient of $\mathbb{P}^1[n]$, and if $g(C) \geq 3$ then $C[n]$ appears as the general fiber of the forgetful morphism $\overline{M}_{g,n} \rightarrow \overline{M}_g$.
We will prove that if either $n \neq 2$ or $\dim(X) \geq 2$, then the connected component of the identity of $Aut(X[n])$ is isomorphic to the connected component of the identity of $Aut(X)$. When $X = C$ is a non-elliptic curve we will classify the dominant morphisms $C[n] \rightarrow C[r]$, and thanks to this we will manage to compute the whole automorphism group of $C[n]$, namely $Aut(C[n]) \cong S_n \times Aut(C)$ for any $n \neq 2$.
Finally, using the techniques developed to deal with the Fulton-MacPherson configuration spaces, we will study the automorphism groups of some Kontsevich moduli spaces $\overline{M}_{0,n}(\mathbb{P}^N, d)$.