



## **Workshop on Mathematical and Computational Problems of Incompressible Fluid Dynamics**

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### **Abstracts**

#### **The Onset of Intermittency in Stochastic Burgers Hydrodynamics**

*G.B Apolinário, L. Moriconi (UFRJ) & R.M. Pereira (UFPE)*

We study the onset of intermittency in stochastic Burgers hydrodynamics, as manifested from the non-Gaussian behavior of large negative velocity gradient fluctuations. Our analysis is carried out within the response functional formalism, where specific velocity configurations - the “viscous instantons” - are assumed to play a dominant role in modeling the left tails of velocity gradient probability distribution functions. We find, as it has been previously conjectured on purely empirical grounds, that the instanton approach becomes meaningful in practice only if the effects of fluctuations around instantons are taken into account. Working with a systematic cumulant expansion, it turns out that the integration of fluctuations yields, in leading perturbative order, to an effective description of the Burgers stochastic dynamics given by the renormalization of its associated heat kernel propagator and the external force-force correlation function.

#### **Chaotic blowup in the 3D incompressible Euler equations on a logarithmic lattice**

*Ciro S. Campolina & Alexei A. Mailybaev, IMPA*

The dispute on whether the three-dimensional (3D) incompressible Euler equations develop infinitely large vorticity in finite time (blowup) keeps increasing due to ambiguous results from state-of-art direct numerical simulations (DNS), while the available simplified models fail to explain the intrinsic complexity and variety of observed structures. Here we propose a new model formally identical to the Euler equations, by imitating the calculus on a 3D logarithmic lattice. This model clarifies the present controversy at the scales of existing DNS and provides the unambiguous evidence of the following transition to the blowup, explained as a chaotic attractor in a renormalized system. The chaotic attractor spans over the anomalously large six-decade interval of spatial scales. For the original Euler system, our results suggest that the existing DNS strategies at the resolution accessible now (and presumably in a rather long future) are unsuitable, by far, for the blowup analysis, and establish new fundamental requirements for the approach to this long-standing problem.

## **Explosive ripple instability due to incipient wave breaking**

*Alexei A. Mailybaev & Andre Nachbin, IMPA*

Considering two-dimensional potential ideal flow with free surface and finite depth, we study dynamics of small-amplitude and short-wavelength wavetrains propagating on the background of a steepening nonlinear wave. This can be seen as a model for small ripples developing on slopes of breaking waves in the surf zone. Using the concept of wave action as an adiabatic invariant, we derive an explicit asymptotic expression for the change of ripple steepness. Through this expression, nonlinear effects are described using intrinsic frequency and intrinsic gravity along Lagrangian (material) trajectories on a free surface. We show that strong compression near the tip on the wave leads to an explosive (super-exponential) ripple instability. This instability may play important role for understanding fragmentation and whitecapping at a surface in breaking waves. Analytical results are confirmed by numerical simulations using a potential theory model.

## **Walking droplets correlated at a distance**

*Andre Nachbin, IMPA*

A hydrodynamic pilot-wave system was discovered a decade ago by Yves Couder and Emmanuel Fort. It takes the form of a millimetric fluid droplet walking on the surface of a vibrating fluid bath. This millimetric droplet can be self-propelled by virtue of a resonant interaction with its own (Faraday) wave field. This system represents the first known example of a pilot-wave system of the form envisaged by Louis de Broglie. Much research has been done studying the dynamics of this wave-particle association. In our computational work we show that two oscillating walkers, confined to separate potential wells, exhibit correlated features even when separated by a large distance. Their phase space dynamics is given by the system as a whole and cannot be described independently. These particles' intricate distributions in phase space are indistinguishable, whereas removing one particle changes completely the phase portrait. The correlated-walkers also relate to nonlinearly-coupled oscillators where synchronization can break out spontaneously. The present oscillator-coupling is dynamic, implicit and mediated through the underlying wave field, as opposed to the Kuramoto model where the phase-coupling scheme is explicit and pre-defined.

## **Energy dissipation caused by boundary layer instability at vanishing viscosity**

*Natacha Nguyen van yen, Matthias Waidmann, Rupert Klein (Freie U. Berlin), Marie Farge (ENS) & Kai Schneider (Aix-Marseille Université)*

A qualitative explanation for the scaling of energy dissipation by high-Reynolds number fluid flows in contact with solid obstacles is proposed in the light of recent mathematical and numerical results. Asymptotic analysis suggests that it is governed by a fast, small-scale Rayleigh–Tollmien–Schlichting instability with an unstable range whose lower and upper bounds scale as  $Re^{3/8}$  and  $Re^{1/2}$ , respectively. By linear superposition, the unstable modes induce a boundary vorticity flux of order  $Re^1$ , a key ingredient in detachment and drag generation according to a theorem of Kato. These predictions are confirmed by numerically solving the Navier–Stokes equations in a two-dimensional periodic channel discretized using compact finite differences in the wall-normal direction, and a spectral scheme in the wall-parallel direction. To download the article: <https://doi.org/10.1017/jfm.2018.396>, <https://zenodo.org/record/1305310>, <https://arxiv.org/abs/1706.00942>

## **A multifractal model for the velocity gradients dynamics in turbulent flows**

*Rodrigo M. Pereira (UFPE), Luca Moriconi (UFRJ) & Laurent Chevillard (ENS de Lyon)*

We develop a stochastic model for the velocity gradients dynamics along a Lagrangian trajectory. Compared to different attempts proposed in the literature, the present model gives a multifractal realistic picture of velocity gradient statistics at any Reynolds number, at the cost of introducing a free parameter known in turbulence phenomenology as the intermittency coefficient. We use as a first modeling step a regularized self-stretching term in the framework of the Recent Fluid Deformation approximation that was shown to give a realistic picture of turbulence only up to moderate Reynolds numbers. As a second step we constrain the dynamics in order to impose a peculiar statistical structure to the dissipation seen by the Lagrangian particle. This probabilistic closure uses as a building block a random field that fulfills the statistical description of the intermittency, i.e. multifractal, phenomenon. These considerations lead us to propose a non-linear and non-Markovian closed dynamics for the elements of the velocity gradient tensor in which (i) the gradients variance is proportional to the Reynolds number, (ii) gradients are typically correlated over the (small) Kolmogorov time scale and gradients norms over the (large) integral time scale (iii) the joint probability distribution function of the two non vanishing invariants  $Q$  and  $R$  reproduces the characteristic teardrop shape, (iv) vorticity gets preferentially aligned with the intermediate eigendirection of the deformation tensor and (v) gradients are strongly non-Gaussian and intermittent.

## **Turbulence of generalized flows in two-dimensions**

*Simon Thalabard, IMPA*

This talk intends to discuss from the point of view of turbulence modeling the concept of generalized flows introduced by Yann Brenier in the 1990's, whereby generalized solutions to the Euler equations are described in terms of a distributional Lagrangian least action principle. In this perspective, one obvious virtue of Brenier's action principle is that it bypasses the deterministic concept of Lagrangian map, and provides a Lagrangian probabilistic framework, which in principle is compatible with the concept of turbulent breakdown of Lagrangian flows. As for now, it is however not clear whether the variational formulation per se indeed produces flows that possess "turbulent features", and/or can prove of any relevance to reproduce inertial-range kinematics as obtained from DNS. To address this issue, we use a statistical mechanics interpretation of Brenier's principle, in order to construct numerically specific instances of two-dimensional generalised flows, and discuss their Eulerian and Lagrangian statistical features.

## **Emergence of skewed non-Gaussian distributions of velocity increments in turbulence**

*W. Sosa-Correa, R. M. Pereira, A. M. S. Macêdo, E. P. Raposo (UFPE),  
D. S. P. Salazar (UFRPE) & G. L. Vasconcelos (UFPR)*

Skewness and non-Gaussian behavior are essential features of the distribution of short-scale velocity increments in turbulent flows. Yet, the physical origin of the asymmetry and the form of the heavy tails remain elusive. Here we describe the emergence of both properties through an exactly solvable stochastic model with a scale hierarchy of energy transfer rates. By a statistical superposition of a local equilibrium distribution weighted by a background density, the increments distribution is given by a novel class of skewed heavy-tailed distributions, written as a generalization of the Meijer-G functions. Nice agreement in the multiscale scenario is found with numerical data of systems with different sizes and Reynolds numbers. Remarkably, the single scale limit provides poor fits to the background density, highlighting the central role of the multiscale mechanism. Our framework can be also applied to describe the challenging emergence of skewed distributions in complex systems.

