Voronoi cells in the Brownian continuum random tree

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Take a uniform random tree with n vertices and select k of those vertices independently and uniformly at random; call them sites. (We assume that n is large and k is fixed, so that with high probability the sites are distinct.) Find the associated Voronoi cells: for each vertex in the tree, assign it to the cell of the site (or sites) which is closest in the graph distance. Now consider the vector of the proportions of the vertices lying in each of the k cells. We prove that this vector converges in distribution to the Dirichlet(1,1,...,1) distribution (that is, it is asymptotically uniform on the (k-1)-dimensional simplex). In fact, this is most easily formulated as a result about the scaling limit of the uniform random tree, namely the Brownian continuum random tree: if we pick k independent sites from the mass measure of the tree, their Voronoi cells have masses which are jointly Dirichlet(1,1,...,1) distributed. An analogue of this result also holds for (the scaling limit of) uniform unicellular random maps on surfaces of arbitrary genus.

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