

Existence of phase transition for percolation on general graphs

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The first step in the study of percolation on a graph G is proving that its critical point $p_c(G)$ for the emergence of an infinite cluster is nontrivial, that is $p_c(G) < 1$. In this talk we prove that, if the isoperimetric dimension of a graph G (with bounded degree) is strictly larger than 4, then $p_c(G) < 1$. This settles a conjecture of Benjamini and Schramm, proposed in their seminal paper [1], saying that $p_c(G) < 1$ for any transitive graph with super-linear growth.

We first prove the existence of an infinite cluster for percolation with random edge-parameters induced by the Gaussian Free Field. Then we integrate out the randomness in the environment by using a multi-scale decomposition of the GFF.

References

- [1] I. BENJAMINI AND O. SCHRAMM , *Percolation beyond \mathbf{Z}^d , many questions and a few answers*, Electronic Communications in Probability (1996)