Convergence analysis of approximation hierarchies for polynomial optimization

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We consider the polynomial optimization problem, which asks to minimize a multivariate polynomial f over a compact semi-algebraic set K. Equivalently, this is asking to find a measure with positive density function, which minimizes the expected value of f over K. This is a hard problem, which has spurred a booming research activity in the past two decades, starting with seminal works by Lasserre and Parrilo in 2000 and onward. In a nutshell, results from real algebraic geometry about positive polynomials and from functional analysis about moments of measures are used to construct hierarchies of bounds that converge to the global minimum of f over K. These bounds are based on using sums-of-squares positivity certificates. While testing positivity of a polynomial is a hard computational problem, the key fact is that there exist efficient algorithms to search for sums of squares of polynomials.

In this lecture we will focus on hierarchies of upper bounds, that are obtained by selecting sums-of-squares density functions with growing degrees d. We will discuss several recent results about the convergence rate of these hierarchies. For general convex bodies K we can show a convergence rate in $O(1/d)$ and, for simpler sets like the hypercube, we can show a stronger convergence rate in $O(1/d^2)$. In addition this convergence analysis is tight, which relies on establishing links to orthogonal polynomials and their extremal roots.

This lecture is based on joint work with Etienne de Klerk.