The Picard group of a tropical toric scheme

Kalina Mincheva[1], Jaiung Jun[2], Jeffrey Tolliver[3]

[1] Yale University

[2] University of Iowa

[3] Akuna Capital

From any monoid scheme X (also known as an $\mathbb{F}_1$-scheme) one can pass to a semiring scheme (a generalization of a tropical scheme) X_S by scalar extension to an idempotent semifield S. We prove that for a given irreducible monoid scheme X (satisfying some mild conditions) and an idempotent semifield S, the Picard group $\mathrm{Pic}(X)$ of X is stable under scalar extension to S (and to any field K). In other words, we show that the groups $\mathrm{Pic}(X)$ and $\mathrm{Pic}(X_S)$ (and $\mathrm{Pic}(X_K)$) are isomorphic. In particular, if $X_{\mathbb{C}}$ is a toric variety, then $\mathrm{Pic}(X)$ is the same as the Picard group of the associated tropical scheme. The Picard groups can be computed by considering the correct sheaf cohomology groups. We also construct the group $\mathrm{CaCl}(X_S)$ of Cartier divisors modulo principal Cartier divisors for a cancellative semiring scheme X_S and prove that $\mathrm{CaCl}(X_S)$ is isomorphic to $\mathrm{Pic}(X_S)$.