Asymptotic Behavior of solutions to a non-homogeneous dissipative perturbation of the Benjamin-Ono equation

Oscar G. Riaño

[1] IMPA, Institudo Nacional de Mátematica Pura e Applicada, Brazil.

We discuss results regarding the large time behavior of solutions to a non-homogeneous dissipative perturbation of the Benjamin-Ono equation,

$$u_t + \gamma \mathcal{H}\partial_{xx} u + \eta(|D|^\alpha + |D|^\beta)u + uu_x = 0,$$

$x \in \mathbb{R}$, $t \geq 0$, $0 < \alpha \leq 1$, $\beta \geq 2$ and the operator $|D|^s$ is defined through its Fourier transform by $\widehat{|D|^s \varphi}(\xi) = |\xi|^s \widehat{\varphi}(\xi)$ for $s > 0$. An example belonging to this class is the Chen-Lee equation ($\alpha = 1$ and $\beta = 2$). This model was first introduced to describe fluid and plasma turbulence for internal waves in a two-fluid system.

In this work we deduce asymptotic expansions for the associated linear problem. For the nonlinear equation with generic initial data, we obtain decays of the solution in several Sobolev norms that correspond to the linearized equation. In particular, global well-posedness and some mixed-norm estimates for the Cauchy problem are established. Finally, we show some asymptotic expansions for solutions.

References

[1] Bona, J. L., and Luo, L. Large-time asymptotics of the generalized Benjamin-Ono-Burgers equation. *Discrete & Continuous Dynamical Systems-S 4*, 1937-1632 (2011), 15.

[2] Chen, H. H., and Lee, Y. C. Nonlinear Dynamical Models of Plasma Turbulence. *Physica Scripta 1982*, T2A (1982), 41–47.

[3] Hayashi, N., and Naumkin, P. I. Asymptotics for the Korteweg–de Vries–Burgers Equation. *Acta Mathematica Sinica 22*, 5 (Sep 2006), 1441–1456.

[4] Karch, G. Large-time behaviour of solutions to non-linear wave equations: higher-order asymptotics. *Mathematical Methods in the Applied Sciences 22*, 18 (1999), 1671–1697.

[5] Vento, S. Asymptotic behavior of solutions to dissipative Korteweg-de Vries equations. *Asymptotic Analysis 68*, 3 (2010), 155–186.