Mass concentration and Lushnikov-like criteria for blow-up for focusing INLS equation

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We consider the initial value problem for the focusing Inhomogeneous Nonlinear Scrhödinger (INLS) Equation

$$\begin{cases} i\partial_t u + \triangle u + |x|^{-b}|u|^{2\sigma}u = 0, & t > 0,\ x \in \mathbb{R}^N, \\ u(\cdot,0) = u_0 \in H^1(\mathbb{R}^N), \end{cases} \tag{1}$$

where $p > 0$ and $b > 0$. Let $s_c = \frac{N}{2} - \frac{2-b}{2\sigma}$. Guzmán showed in [4] that the problem (1) is well-posed in $H^1(\mathbb{R}^N)$ for $0 < \sigma < \frac{2-b}{N-2}$ and $0 < b < \widetilde{2}$, where $\widetilde{2} = \frac{N}{3}$ if $N = 1, 2, 3$ and $\widetilde{2} = 2$ if $N \geq 4$.

We show the following result of mass concentration

Theorem 1: *Suppose $s_c = 0$ and $0 < b < \widetilde{2}$. Let u solution for (1) which blows-up at time finite $T > 0$, and $\lambda(t) > 0$ any function such that $\lambda(t)\|\nabla u\|_{L_2} \to \infty$ as $t \uparrow T$. Then, there exists $x(t) \in \mathbb{R}^N$ such that*

$$\liminf_{t\uparrow T} \int_{|x-x(t)|\leq\lambda(t)} |u(x,t)|^2\, dx \geq \|Q\|^2_{L_2}.$$

Let $0 < s_c < 1$ and $b < min\{2, N\}$, then the Gagliardo-Nirenberg inequality

$$\int_{\mathbb{R}^N} |x|^{-b}|u(x)|^{2\sigma+2}\, dx \leq C_{\sigma,N}\|\nabla u\|^{N\sigma+b}_{L^2(\mathbb{R}^N)}\|u\|^{2\sigma+2-(N\sigma+b)}_{L^2(\mathbb{R}^N)} \tag{2}$$

holds, where $C_{\sigma,N} > 0$ is the sharp constant, see [2]. Also we obtain Lushnikov's criteria for PVI for

Theorem 2: *Suppose that $u_0 \in H^1(\mathbb{R})$ and $V(0) < \infty$. The following is a sufficient condition for blow-up in finite time for (1) with $s_c > 0$ and $E[u] > 0$:*

$$\frac{V_t(0)}{M} < \sqrt{8Ns_c}g\left(\frac{4}{Ns_c}\frac{EV(0)}{M^2}\right), \tag{3}$$

where

$$g(x)=\begin{cases}\sqrt{\frac{1}{kx^k}+x-(1+\frac{1}{k})} & \textit{if } 0<x\leq 1\\ -\sqrt{\frac{1}{kx^k}+x-(1+\frac{1}{k})} & \textit{if } x\geq 1\end{cases}\quad \textit{with } k=\frac{(p-1)s_c}{2}. \tag{4}$$

Theorem 3: *Suppose that $u_0 \in H^1$ and $V(0)<\infty$. The following is a sufficient condition for blow-up in finite time for INLS* (1) *with $s_c>0$ and $E[u]>0$:*

$$\frac{V_t(0)}{M}<\frac{4\sqrt{2}M^{\frac{1}{2}-\frac{p+1}{N(p-1)+2b}}E^{\frac{s_c}{N}}}{C}g\left(C^2\frac{E^{\frac{4}{N(p-1)+2b}}V(0)}{M^{1+\frac{2(p+1)}{N(p-1)+2b}}}\right) \tag{5}$$

where g is defined in (4)*,*

$$C=\left(\frac{2(p+1)}{s_c(p-1)}(C_{p,N})^{\frac{N(p-1)+2b}{2}+(p+1)}\right)^{\frac{2}{N(p-1)+2b}}. \tag{6}$$

and $C_{p,N}$ is a sharp constant in the interpolation inequality (2)*.*

References

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