HEIGHT ESTIMATES FOR BIANCHI GROUPS

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ABSTRACT

Consider the geometry of the action of Bianchi groups $\mathrm{SL}(2, \mathcal{O}_d)$ on the hyperbolic space $\mathbb{H}^3$, where $\mathcal{O}_d$ is the ring of integers of the imaginary quadratic field $K = \mathbb{Q}(\sqrt{-d})$. We obtain, for some values of d, a height estimate $H(M) \leq cD(z,t)^9$, for some matrix M that take a given point $(z,t) \in \mathbb{H}^3$ into the fundamental domain of the Bianchi group. Here, c is a constant that does not depend on the point and $D(z,t)$ is an explicit function of the coordinates of the initial point. This generalizes a lemma of Habegger and Pila about the action of the modular group on $\mathbb{H}^2$.