Shadowing properties of group actions

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Let Φ be a uniformly continuous action of a finitely generated group G on a metric space.

The shadowing property of Φ means that, given an approximate trajectory, we can find an exact trajectory close to it. The Reductive Shadowing Theorem (RST) states that if the action of a one-dimensional subgroup of G is topologically Anosov (i.e., it has the shadowing property and is expansive), then the action Φ is topologically Anosov as well (and hence, Φ has the shadowing property).

The first RST was proved in [1] for the groups $\mathbb{Z}^p$; later it was proved for virtually nilpotent groups [2]. At the same time, it was shown in [2] that the RST is not valid, for example, for the Baumslag–Solitar groups $\mathrm{BS}(1, n)$ with $n > 1$.

The inverse shadowing property of Φ means that, given a family of approximate trajectories (generated by a so-called approximate method), for any fixed exact trajectory of Φ, we can find a member of this family that is close to this fixed trajectory. It is shown in [3] that an analog of the RST for the case of inverse shadowing (with “topologically Anosov” replaced by the so-called “Tube Condition”) is also valid for virtually nilpotent groups.

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[3] S.Yu.Pilyugin, Inverse shadowing in group actions, *Dyn. Syst.*, **32** (2017), 198-210.