Hausdorff dimension for projections of dynamically defined Cantor sets

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A classical theorem of Marstrand states that for any Borel subsets $F_1, F_2 \subset \mathbb{R}$

$$HD(F_1 + \lambda \cdot F_2) = \min\{1, HD(F_1) + HD(F_2)\},$$

for Lebesgue almost all $\lambda \in \mathbb{R}$. Moreira was able to improve this theorem for dynamically defined Cantor sets. He proved that given two such sets $K_1, K_2 \subset \mathbb{R}$, satisfying some generic hypothesis, one has

$$HD(K_1 + \lambda \cdot K_2) = \min\{1, HD(K_1) + HD(K_2)\},$$

for all $\lambda \neq 0$. We will talk about how Moreiras ideas can be generalized to Cantor sets in the complex plane. We will have a similar formula which holds for dynamically defined complex Cantor sets. In particular, this Cantor sets include Julia sets associated to quadratic maps $Q_c(z) = z^2 + c$ when the parameter c is not in the Mandelbrot set.