Periodic billiard orbits in obtuse triangles

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The existence of periodic billiard orbits in polygons is an open problem in mathematics. Even for a triangle there is still no answer. For acute triangles the answer is well known since the triangle whose vertices are the base points of the three altitudes of the triangle is a periodic orbit. For obtuse triangles, in general, little is known. The aim of this work is to collect results and techniques on periodic billiard orbits in obtuse triangles. All cases covered include a wide variety of triangles, but the question of the existence of periodic billiard orbits for all triangles is far from being fully contemplated.

1 Results

We start by introducing the work of Vorobets, Gal'perin and Stepin, who unified in the early 1990s the known cases of triangles that have periodic billiard orbits, introduced the concept of stability and proved new results, such as an infinite family of stable orbits. We also have the theorem of Halbeisen and Hungerbühler of 2000 extending the families of stable orbits.

Next, we mention the works of Schwartz of 2006 and 2009 that use computational assistance to prove that every triangle whose angles are at most 100° have periodic billiard orbits. Then, we have the results of 2008 by Hooper and Schwartz on periodic billiard orbits in nearly isosceles triangles and on stability of billiard orbits in Veech triangles.

There is also a result by Hooper which gives a homological condition to existence of billiard periodic orbits on triangles.

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