On one generalization of the skew tent map

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The one dimensional map $f_v : [0,1] \to [0,1]$, whose graph linearly extends points $(0,0)$ to $(v,1)$ to $(1,0)$, where $v \in (0,1)$ is a parameter, is called skew tent map. It was proved in [1] that any skew tent map is topologically conjugated to the tent map $x \mapsto 1 - |1-2x|$. For any $x^* \in (0,1)$ and $n \geq 1$ define $x_n^- = \min\{x^*; \max\{x < x^* : f^n(x) = 0\}\}$ and $x_n^+ = \max\{x^*; \min\{x > x^* : f^n(x) = 0\}\}$. Denote $d_n = \frac{h(x_n^+)-h(x_n^-)}{x_n^+ - x_n^-}$. We will present the next results:

Theorem 1 [3]: The derivative of the conjugacy of f_v and the tent map at $x^* \in [0,1]$ exists if and only if there exists $\lim\limits_{n\to\infty} d_n$.

Theorem 1 for $x^* \in \mathbb{Q}$ was also proved in [2]. Call a piecewise linear unimodal map $g : [0,1] \to [0,1]$ a carcass map, if all its kinks belong to the complete pre-image of 0.

Theorem 2: Any carcass map is topologically conjugated to the tent map.

Theorem 3: Let h be the conjugacy of a carcass map and the tent map. If $0 < h'(x^*) < \infty$ for a x^*, then h is piecewise linear.

We can use Theorems 2 and 3 to construct counterexamples, which show impossibility to generalize Theorem 1 to carcass maps.

References

[1] Skufca J. and Bollt E., *A concept of homeomorphic defect for defining mostly conjugate dynamical systems*, Chaos, 2008

[2] Plakhotnyk M., *Differentiability of the homeomorphism of conjugateness for the pair of tent-like interval itself maps*, Matematychni Studii, Vol. 46, No 2, pp. 196-202, 2016.

[3] Plakhotnyk M., *The derivative of the conjugacy between skew tent maps*, International Journal of Bifurcation and Chaos in Applied Sciences and Engineering. *Accepted for publication 13.06.2018.*