Exponential decay properties of solutions to the fifth-order KdV equation

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We consider the Cauchy problem for the fifth-order nonlinear dispersive equation of the form

$$\partial_t u - \partial_x^5 u + Q(u, \partial_x u, \cdots, \partial_x^3 u) = 0,\ u = u(x,t),\ x \in \mathbb{R}\,,\ t \in [0,T],$$
$$u(x,0) = u_0(x).$$

where Q is a polynomial. We prove that if the initial data u_0 is such that $e^{a_0 x_+^{5/4}} u_0 \in L^2(\mathbb{R})$, then the solution $u(\cdot,t)$ of the problem has also the same type of decay as time evolves, but the strength of the decay decreases in time.

More precisely,

$$\| e^{a(t) x_+^{5/4}} u(\cdot,t) \|_{L^2(\mathbb{R})} \leq C \quad \text{for } t \in [0,T],$$

where

$$a(t) = \frac{a_0}{(1 + \frac{5^5}{4^3}\, a_0^4\, t)^{1/4}}.$$

We also prove that this decay is the best that can be expected.