Geodesic submanifolds of hyperbolic hybrids

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Let M be a finite-volume hyperbolic n-manifold, $n > 2$, with fundamental group Γ. Mostow rigidity and the Margulis commensurator theorem imply that arithmeticity of Γ is equivalent to Γ having infinite index in its abstract commensurator.

This group-theoretic characterization of arithmeticity has geometric consequences: if M contains a properly immersed totally geodesic hypersurface, then it contains infinitely many and they are everywhere dense. Reid and McMullen independently asked whether this geometric property implies arithmeticity, that is, if M contains infinitely many totally geodesic hypersurfaces then is Γ necessarily arithmetic?

I will explain progress toward a positive solution to this question, which is joint work with D. Fisher, J.-F. Lafont, and N. Miller. We proved that many nonarithmetic hyperbolic manifolds constructed from cut-and-paste methods, e.g., all the famous examples of Gromov and Piatetski-Shapiro, have the property that the set of maximal totally geodesic hypersurfaces is finite. These produce the first examples of hyperbolic n-manifolds for which the collection of geodesic hypersurfaces is known to be finite but nonempty. I will also discuss the generalization to higher codimension.