# Invariance principle for the random Lorentz gas beyond the [Boltzmann-Grad / Gallavotti-Spohn] limit


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Let hard ball scatterers of radius $r$ be placed in $\mathbb{R}^d$, centred at the points of a Poisson point process of intensity $\rho$. The volume fraction $r^d\rho$ is assumed to be sufficiently low so that with positive probability the origin is not trapped in a finite domain fully surrounded by scatterers. The Lorentz process is the trajectory of a point-like particle starting from the origin with randomly oriented unit velocity subject to elastic collisions with the fixed (infinite mass) scatterers. The question of diffusive scaling limit of this process is a major open problem in classical statistical physics.

Gallavotti (1969) and Spohn (1978) proved that under the so-called Boltzmann-Grad limit, when $r \to 0$, $\rho \to \infty$ so that $r^{d-1}\rho \to 1$ and the time scale is fixed, the Lorentz process (described informally above) converges to a Markovian random flight process, with independent exponentially distributed free flight times and Markovian scatterings. It is essentially straightforward to see that taking a second diffusive scaling limit (after the Gallavotti-Spohn limit) yields invariance principle.

I will present new results going beyond the [Boltzmann-Grad / Gallavotti-Spohn] limit, in $d = 3$: Letting $r \to 0$, $\rho \to \infty$ so that $r^{d-1}\rho \to 1$ (as in B-G) and *simultaneously* rescaling time by $T \sim r^{-2+\epsilon}$ we prove invariance principle (under diffusive scaling) for the Lorentz trajectory. Note that the B-G limit and diffusive scaling are done simultaneously and not in sequel. The proof is essentially based on control of the effect of re-collisions by probabilistic coupling arguments. The main arguments are valid in $d = 3$ but not in $d = 2$.

Joint work with Chris Lutsko (Bristol)