Minimal mass blow up solutions for critical nonlinear dispersive problems

Yvan Martel [1]

[1] CMLS, Ecole polytechnique, CNRS, France

In this talk, I plan to review several results of existence of minimal mass blow up solutions for several mass critical nonlinear dispersive problems. Such minimal mass solutions concentrate all their mass at blow up points. Such problems are thus doubly critical : both for the nonlinearity whose power is the lowest one for which blow up is possible, and for the amount of mass of the solution.

I will first recall such pioneering results by Merle and by Raphaël-Szeftel. Then, I will present recent results obtained in collaboration with Merle-Raphaël, Le Coz-Raphaël, Combet, and Pilod on several nonlinear dispersive models.

References

[1] V. Combet and Y. Martel, *Sharp asymptotics for the minimal mass blow up solution of critical gKdV equation*, Bulletin des Sciences Mathématiques, Elsevier, 2017, 141 (2), pp.20 - 103.

[2] V. Combet and Y. Martel, *Construction of multi-bubble solutions for the critical gKdV equation,* SIAM Journal on Mathematical Analysis, Society for Industrial and Applied Mathematics, In press.

[3] S. Le Coz, Y. Martel and P. Raphaël, *Minimal mass blow up solutions for a double power nonlinear Schrödinger equation,* Revista Matemática Iberoamericana, European Mathematical Society, 2016, 32 (3), pp.795-833.

[4] Y. Martel, *Asymptotic N-soliton-like solutions of the subcritical and critical generalized Korteweg de Vries equations,* Amer. J. Math., **127** (2005), no. 5, 1103-1140.

[5] Y. Martel, F. Merle and P. Raphaël, *Blow-up for the critical generalized Korteweg-de Vries equation I: dynamics near the soliton,* Acta Math., **212** (2014), 59-140.

[6] Y. Martel, F. Merle and P. Raphaël, *Blow-up for the critical generalized Korteweg-de Vries equation II: minimal mass dynamics,* J. Eur. Math. Soc., **17** (2015), 1855-1925.

[7] Y. Martel and D. Pilod, *Construction of a minimal mass blow up solution of the modified Benjamin–Ono equation,* Mathematische Annalen, Springer Verlag, 2017, 369 (1-2), pp.153-245.

[8] F. Merle, *Construction of solutions with exactly k blow-up points for the Schrödinger equation with critical nonlinearity,* Comm. Math. Phys, **129** (1990), 223-240.

[9] P. Raphaël and J. Szeftel, *Existence and uniqueness of minimal blow-up solutions to an inhomogeneous mass critical NLS,* J. Amer. Math. Soc., **24** (2011), 471-546.