Gravitational allocation to uniform points on the sphere

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Given n uniform points on the surface of a two-dimensional sphere, how can we partition the sphere fairly among them ?

"Fairly" means that each region has the same area. It turns out that if the given points apply a two-dimensional gravity force to the rest of the sphere, then the basins of attraction for the resulting gradient flow yield such a partition—with exactly equal areas, no matter how the points are distributed. (See the cover of the AMS Notices at http://www.ams.org/publications/journals/notices/201705/rnoticvr1.pdf .) Our main result is that this partition minimizes, up to a bounded factor, the average distance between points in the same cell. I will also present an application to almost optimal matching of n uniform blue points to n uniform red points on the sphere, connecting to a classical result of Ajtai, Komlos and Tusnady (Combinatorica 1984). Joint work with Nina Holden and Alex Zhai. I will also state open problems on greedy and electrostatic bipartite matching.