

Subsets of posets minimising the number of chains

Wojciech Samotij ¹,

¹ Tel Aviv University

A well-known theorem of Sperner describes the largest collections of subsets of an n -element set none of which contains another set from the collection. Generalising this result, Erdos characterised the largest families of subsets that do not contain a chain of sets of an arbitrary length k . The extremal families contain all subsets whose cardinalities belong to an interval of length $k-1$ centred around $n/2$. In a far-reaching extension of Sperner's theorem, Kleitman determined the smallest number of chains of length two that have to appear in a collection of a given number a of subsets. For every a , this minimum is achieved by the collection comprising a sets whose cardinalities are as close to $n/2+1/4$ as possible. Kleitman conjectured that the same is true about chains of an arbitrary length k , for all a and n . We will sketch a proof of this conjecture.